Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel Level 3 GCE

Centre Number	Candidate Number

Thursday 20 June 2019

Morning (Time: 1 hour 30 minutes)	Paper Reference **9FM0/3C**

Further Mathematics

Advanced

Paper 3C: Further Mechanics 1

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Unless otherwise indicated, whenever a numerical value of g is required, take $g = 9.8\,\text{m}\,\text{s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 7 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

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Pearson

1.

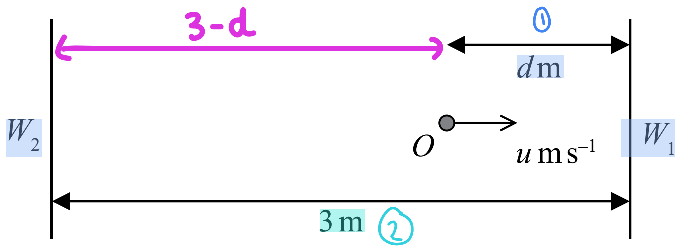


Figure 1

Figure 1 represents the plan of part of a smooth horizontal floor, where W_1 and W_2 are two fixed parallel vertical walls. The walls are 3 metres apart. ②

A particle lies at rest at a point O on the floor between the two walls, where the point O is d metres, $0 < d \leqslant 3$, from W_1

①

At time $t = 0$, the particle is projected from O towards W_1 with speed u m s^{-1} in a direction perpendicular to the walls.

The coefficient of restitution between the particle and each wall is $\frac{2}{3}$

③ ②

The particle returns to O at time $t = T$ seconds, having bounced off each wall once.

(a) Show that $T = \dfrac{45 - 5d}{4u}$

(6)

The value of u is fixed, the particle still hits each wall once but the value of d can now vary.

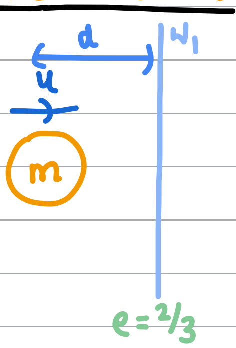
(b) Find the least possible value of T, giving your answer in terms of u. You must give a reason for your answer.

(2)

(a) recognising this as a 'successive direct impacts' question (involving a wall) - hence need to consider each section of the particle's path SEPARATELY (preferably with separate diagrams)

• firstly: section up to particle collision with W_1 - ①

BEFORE collision 1



working out the time taken for particle to travel 'd' m at 'u' ms^{-1} using speed = $\frac{\text{distance}}{\text{time}}$ rearranged

time = $\frac{\text{distance}}{\text{speed}}$

$\boxed{\Rightarrow \text{time} = \frac{d}{u}}$

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Question 1 continued

• next, section from after collision with W_1 to before collision with W_2 –②

first need to work out the velocity of the particle AFTER collision 1

AFTER collision 1

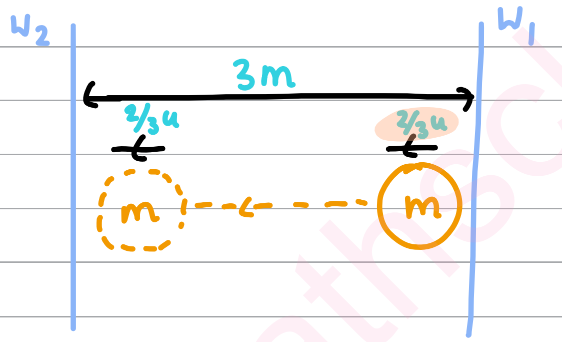
subbing into impact law – formula for 'e':

$$e = \frac{\text{speed of separation}}{\text{speed of approach}} = \frac{v}{u}$$

$$\frac{2}{3} = \frac{v}{u}$$

$\times u$ $\times u$

$$\Rightarrow v = \frac{2}{3}u$$



working out the time taken for the particle to travel '3'm at $\frac{2}{3}u$ ms^{-1}

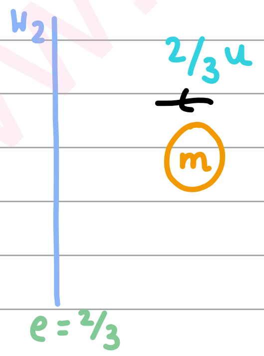
$$\text{time} = \frac{\text{distance}}{\text{speed}}$$

$$\boxed{\text{time} = \frac{3}{^2/_3 u}}$$

– finally – section AFTER collision 2 to 'O'

first need to consider 'before' and 'after' of collision 2 and work out the velocity of the particle AFTER it

BEFORE collision 2



AFTER collision 2

subbing into impact law – formula for 'e'

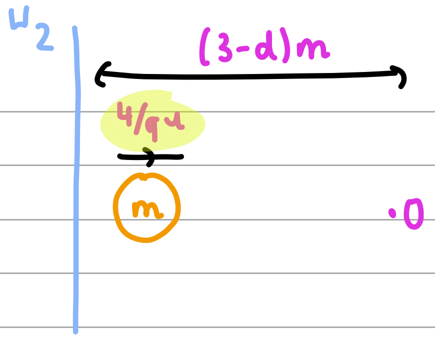
$$e = \frac{v}{u}$$

$$\frac{2}{3} = \frac{v}{^2/_3 u}$$

$\times ^2/_3 u$ $\times ^2/_3 u$

$$\Rightarrow v = \frac{4}{9}u$$

Question 1 continued



finding the time taken for particle to travel $(3-d)$m at $4/9\ ms^{-1}$

$$\text{time} = \frac{\text{distance}}{\text{speed}}$$

$$\boxed{\text{time} = \frac{3-d}{4/9u}}$$

$\therefore$ Summing all the sections' TIMES to get total time for the particle to return to O

$$T = \frac{d}{u} + \frac{3}{2/3u} + \frac{3-d}{4/9u}$$

Simplifying fractions

$$\Rightarrow T = \frac{d}{u} + \frac{9}{2u} + \frac{9(3-d)}{4u}$$

getting common denominator

$$T = \frac{4d}{4u} + \frac{2(9)}{4u} + \frac{9(3-d)}{4u}$$

$$\therefore T = \frac{4d + 18 + 27 - 9d}{4u}$$

$$\boxed{\Rightarrow T = \frac{45-5d}{4u}}$$

(b) looking at the expression for T in part (a) and realising that to minimise T need to max. d (to get smaller numerator)

↳ the interval for 'd' is $0 < d \leq 3$ so $d_{max} = 3$ - sub this '3' into the expression for T

$$T_{min.} = \frac{45-5(3)}{4u} = \frac{\cancel{30}^{15}}{\cancel{4}_{2}u} = \boxed{\frac{15}{2u}\ \text{secs}}$$

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Question 1 continued

(Total for Question 1 is 8 marks)

2.

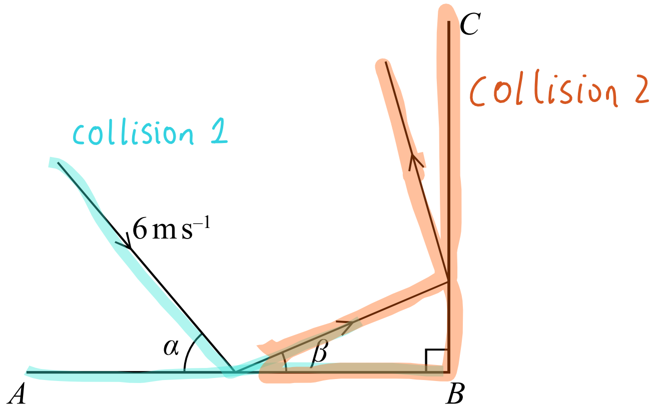


Figure 2

Figure 2 represents the plan view of part of a horizontal floor, where AB and BC are fixed vertical walls with AB perpendicular to BC.

A small ball is projected along the floor towards AB with speed $6\,\text{m s}^{-1}$ on a path that makes an angle α with AB, where $\tan\alpha = \frac{4}{3}$. The ball hits AB and then hits BC.

Immediately after hitting AB, the ball is moving at an angle β to AB, where $\tan\beta = \frac{1}{3}$

The coefficient of restitution between the ball and AB is e.

The coefficient of restitution between the ball and BC is $\frac{1}{2}$

By modelling the ball as a particle and the floor and walls as being smooth,

(a) show that the value of $e = \frac{1}{4}$ **(5)**

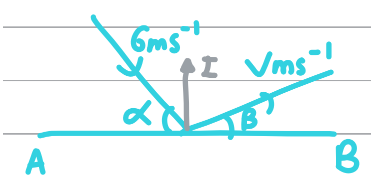
(b) find the speed of the ball immediately after it hits BC. **(4)**

(c) Suggest two ways in which the model could be refined to make it more realistic. **(2)**

recognising this as a successive oblique impacts question ∴ know need to consider each collision separately

(a) let's first consider the first collision – the one between the small ball and AB

FIRST COLLISION



...perpendicular :

remembering how when a particle collides obliquely with a fixed surface, the IMPULSE acts perpendicular to the plane of impact ∴ only the perpendicular components

P 6 2 6 7 4 A 0 6 2 8

Question 2 continued

Of the $v=6$ CHANGE – here NEL rearranged applies

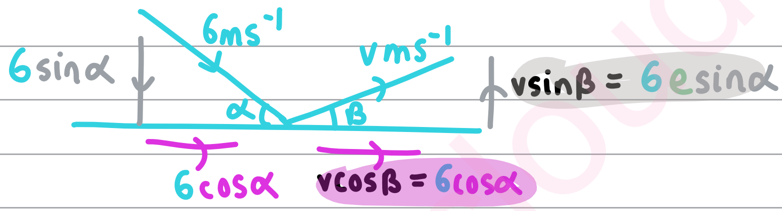
$\Rightarrow e6\sin\alpha = v\sin\beta$

...parallel:

doesn't change ∴ can just resolve the $v=6$ using trig

$\Rightarrow 6\cos\alpha = v\cos\beta$

...adding the resolved components onto the diagram:



where $\tan\alpha = 4/3$ and $\tan\beta = 1/3$

...now, two ways to proceed:

METHOD 1: using trig expressions (faster!)

we can use the fact that we know the value of $\tan\beta = 1/3$

–using above diagram: $\tan\beta = \frac{v\sin\beta}{v\cos\beta} = \frac{\cancel{6}e\sin\alpha}{\cancel{6}\cos\alpha}$

$\Rightarrow \tan\beta = e\tan\alpha$

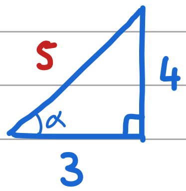
$\frac{1}{3} = e\left(\frac{4}{3}\right)$

$\div 4/3 \quad \div 4/3$

$\Rightarrow \boxed{e = \frac{1}{\cancel{3}} \times \frac{\cancel{3}}{4} = \frac{1}{4}}$

METHOD 2: using exact numerical values for the trig expressions

from $\tan\alpha = 4/3$ we can draw a right-angled triangle and use the Pythag. triple 3-4-5 to find values for $\sin\alpha$ and $\cos\alpha$



$\Rightarrow \sin\alpha = O/H = 4/5$

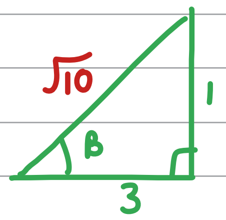
$\cos\alpha = A/H = 3/5$

$\tan\alpha = 4/3$ ← given

and likewise for $\tan\beta = 1/3$ – using Pythagoras'

Question 2 continued

$$\sqrt{(1)^2+(3)^2} = \sqrt{10}$$

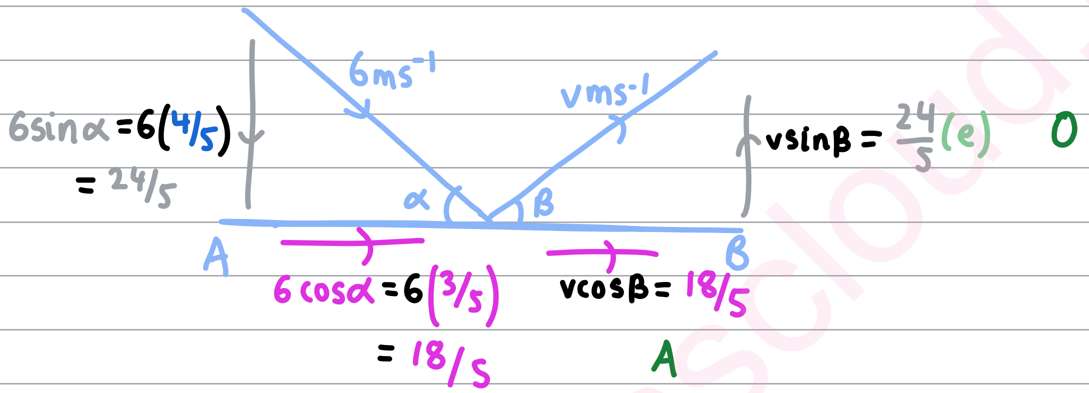


$\Rightarrow$ $\sin\beta = O/H = 1/\sqrt{10}$

$\cos\beta = A/H = 3/\sqrt{10}$

$\tan\beta = 1/3$ ← given

Subbing these into prev. diagram:



WAY 1: using fact that $\tan\beta = 1/3$

i.e evaluating $\tan\beta = O/H$ now using numerical values

$$\frac{\frac{24}{5}e}{18/5} = 1/3$$

$\times\frac{18}{5}$ $\qquad\qquad$ $\times\frac{18}{5}$

$$\Rightarrow \frac{24}{5}e = \frac{6}{5}$$

$\div\frac{24}{5}$ $\qquad\qquad$ $\div\frac{24}{5}$

$$e = \frac{\cancel{6}^{1}}{\cancel{5}}\left(\times\frac{\cancel{5}}{\cancel{24}_{4}}\right)$$

$$\Rightarrow \boxed{e = 1/4}$$

WAY 2: using the velocity components:

...first perpendicular:

$$6e\sin\alpha = v\sin\beta$$

$\div\sin\alpha$ $\qquad\qquad$ $\div\sin\alpha$

$$\Rightarrow e = \frac{v\sin\beta}{6\sin\alpha}$$ but need 'v' – ①

can get from the parallel components:

$$6\cos\alpha = v\cos\beta$$

$$\div\cos\beta \quad \div\cos\beta$$

$$v = \frac{6\cos\alpha}{\cos\beta} = \frac{6\left(3/5\right)}{3/\sqrt{10}}$$

$$\Rightarrow v = \frac{6\sqrt{10}}{5} \quad -②$$

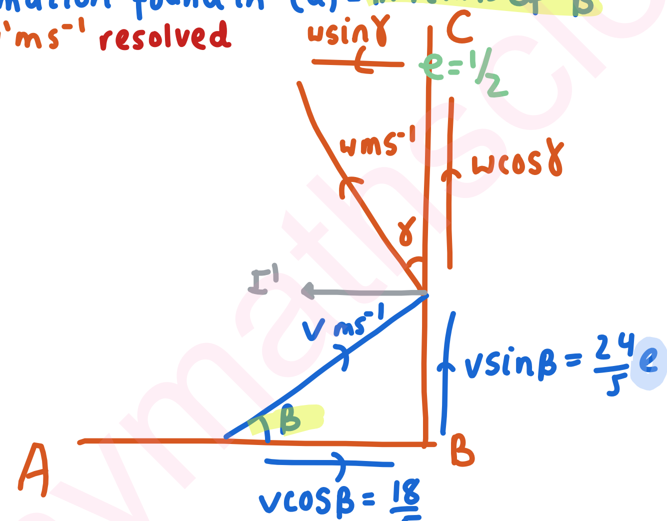
Subbing ② into ①

$$e = \frac{\left(\frac{6\sqrt{10}}{5}\right)\left(1/\sqrt{10}\right)}{6\left(4/5\right)} = \frac{\frac{6}{5}}{\frac{24}{5}} = \frac{\cancel{6}^{1}}{\cancel{24}_{4}} = \frac{1}{4}$$

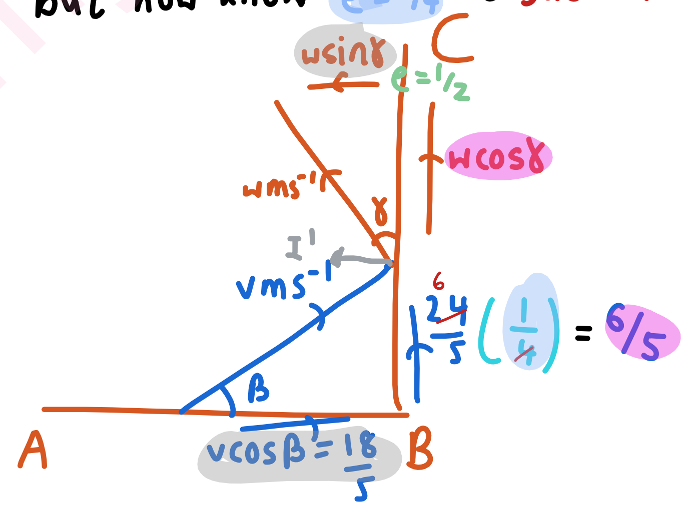
$$\boxed{\therefore e = 1/4}$$

(b) now focusing on the second collision of the small ball - the one it makes with BC

- in blue is the information found in (a) - IN TERMS OF 'β'
- in orange is the 'w' ms^{-1} resolved



but now know $e = 1/4$ so sub it into above diagram



now ctd. from (a) - we're interested in finding the speed of the small ball after this second collision - the one with wall BC - call it 'w'

WAY 1: using properties of angle β

Now we see how the wall BC becomes the fixed surface that the impulse I perpendicular to:

...first, perpendicular components:

Only these CHANGE - NEL rearranged applies:

$$ev\cos\beta = w\sin\gamma$$

$$\underbrace{\tfrac{1}{2}\left(\tfrac{18}{5}\right)}_{\text{from DIAGRAM}} \quad \text{OR} \quad \underbrace{\tfrac{1}{2}\left(\tfrac{6\sqrt{10}}{5}\right)\left(\tfrac{3}{\sqrt{10}}\right)}_{\text{from exact values in part (a) METHOD 2 and 'v' in METHOD 2 WAY 2}} = w\sin\gamma$$

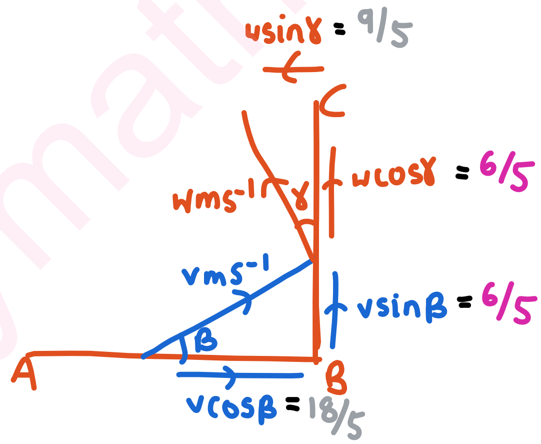
$$\Rightarrow \tfrac{9}{5} = w\sin\gamma$$

...next, for parallel components - these remain the same:

$$v\sin\beta = \underbrace{\left(\tfrac{6\sqrt{10}}{5}\right)\left(\tfrac{1}{\sqrt{10}}\right)}_{\text{using exact values}} \text{ or } \underbrace{\tfrac{6}{5}}_{\text{diagram}} = w\cos\gamma$$

$$\Rightarrow \tfrac{6}{5} = w\sin\gamma$$

Subbing these into diagram



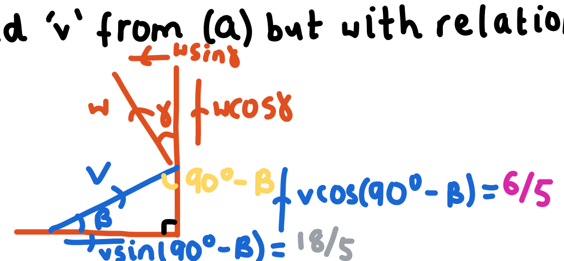
SO now that we have the components of 'w' - let's pythagorise to get $|w|$

$$|w| = \sqrt{\left(\tfrac{6}{5}\right)^2 + \left(\tfrac{9}{5}\right)^2}$$

$$\boxed{= \tfrac{3\sqrt{13}}{5}\ \text{ms}^{-1}}$$

WAY 2: we can use the angle $(90° - \beta)$ and exact values

populating resolved 'v' from (a) but with relation to the $(90-\beta)°$ angle



Question 2 continued

... perpendicular to BC after 2nd collision (NEL rearranged applies)

$$ev\sin(90°-\beta) = u\sin\gamma$$

which following corresponding angles rule: $\sin(90°-\beta) = \cos\beta$

$$ev\cos\beta = u\sin\gamma$$

$$\underbrace{\tfrac{1}{2}\left(\tfrac{18}{5}\right)}_{\text{diagram}} \text{ OR } \underbrace{\tfrac{1}{2}\left(\tfrac{6\sqrt{10}}{5}\right)\left(\tfrac{3}{\sqrt{10}}\right)}$$

from exact values in part (a) METHOD 2 and 'v' in METHOD 2 WAY 2

$$\Rightarrow \frac{9}{5} = u\sin\gamma$$

... next, for parallel components - these remain the same:

$$v\cos(90°-\beta) = u\cos\gamma$$

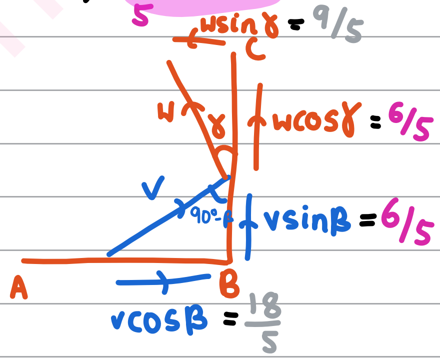
which following the corresponding angles rules:

$$\cos(90°-\beta) = \sin\beta$$

$$v\sin\beta = u\cos\gamma$$

$$\underbrace{\tfrac{6}{5}}_{\text{diagram}} \text{ or } \underbrace{\left(\tfrac{6\sqrt{10}}{5}\right)\left(\tfrac{1}{\sqrt{10}}\right)}_{\text{using exact values}} = u\cos\gamma$$

$$\Rightarrow \frac{6}{5} = u\sin\gamma$$



so now that we have the COMPONENTS of 'u' - let's pythagorise to get $|u|$

$$|u| = \sqrt{(6/5)^2 + (9/5)^2}$$

$$\boxed{= \frac{3\sqrt{13}}{5}\text{ms}^{-1}}$$

(c) 'modelling the ball as a particle' - from Chp 8 Yr 1 Mechanics:
- consider air resistance
- spin/rotation
- give BALL DIMENSIONS

'smooth walls' - include friction between floor and ball
- include friction balls + walls

(Total for Question 2 is 11 marks)

3. A particle P, of mass 0.5 kg, is moving with velocity $(4\mathbf{i} + 4\mathbf{j})\text{ m s}^{-1}$ when it receives an impulse $\mathbf{I}$ of magnitude 2.5 Ns.

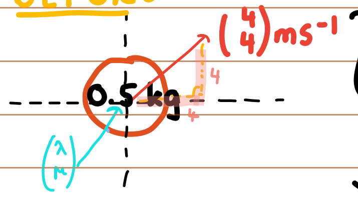
As a result of the impulse, the direction of motion of P is deflected through an angle of 45°

Given that $\mathbf{I} = (\lambda\mathbf{i} + \mu\mathbf{j})$ Ns, find all the possible pairs of values of λ and μ.

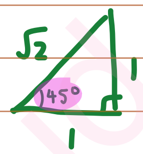
(9)

...first starting with a detailed diagram (keeping the vector notation):

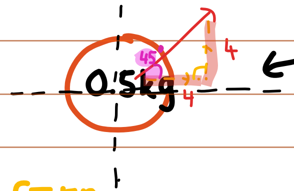
BEFORE:



useful thing to notice here is that the $\binom{4}{4}$ velocity triangle is a scalar multiple of the exact value triangle:



∴ know that the particle is travelling at an angle 45° to the horizontal



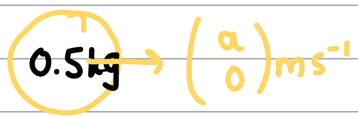
AFTER:

hence, if the particle is deflected by 45° then that means the angle that it makes after receiving the impulse makes either

option 1: 45° − 45° = 0° with the horizontal

⇒ moving directly to the right with an unknown velocity BUT MOMENTUM is $\binom{a}{0}$:

WAY 1: reflecting this momentum in the velocity AFTER also being $\binom{a}{0}$ms^{-1}

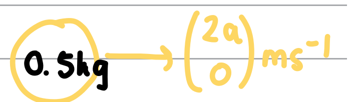


WAY 2: subbing this momentum into formula for momentum:

$p = mv$

momentum (kg/ms^{-1}), mass (kg), velocity (ms^{-1})

$\binom{a}{0} = 0.5v \Rightarrow v = \binom{2a}{0}$ms^{-1}



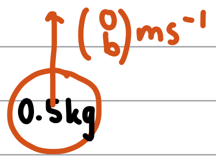
Question 3 continued

option 2 : $45° + 45° = 90°$ with the horizontal

$\Rightarrow$ moving directly upwards with an unknown velocity but MOMENTUM is $\begin{pmatrix}0\\b\end{pmatrix}$

WAY 1: reflecting this momentum in the velocity AFTER also being $\begin{pmatrix}0\\b\end{pmatrix} ms^{-1}$

AFTER :



WAY 2: subbing this momentum into formula for momentum :

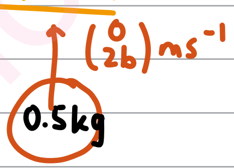
$$p = mv$$

$$\begin{pmatrix}0\\b\end{pmatrix} = 0.5v$$

$\div 0.5$ $\quad$ $\div 0.5$

$$\Rightarrow v = \begin{pmatrix}0\\2b\end{pmatrix} ms^{-1}$$

AFTER :



Let's evaluate both option 1 and option 2 into the vector formula for impulse: $\underline{I} = m(\underline{v} - \underline{u})$

using WAY 1:

...first option 1 :

$$\underline{I} = m(\underline{v} - \underline{u})$$

$$\begin{pmatrix}\lambda\\\mu\end{pmatrix} = 0.5\left(\begin{pmatrix}a\\0\end{pmatrix} - \begin{pmatrix}4\\4\end{pmatrix}\right)$$

$$\Rightarrow \begin{pmatrix}\lambda\\\mu\end{pmatrix} = 0.5\begin{pmatrix}a-4\\-4\end{pmatrix}$$

factor the 0.5 into bracket

$$\Rightarrow \begin{pmatrix}\lambda\\\mu\end{pmatrix} = \begin{pmatrix}0.5a-2\\-2\end{pmatrix}$$

$\Rightarrow \lambda = 0.5a - 2$ -①

$\mu = -2$ -②

and using fact that the magnitude of $\underline{I}$ is 2.5 Ns (Pythagoras')

$$(\lambda)^2 + (\mu)^2 = (2.5)^2$$

$\Rightarrow \lambda^2 + \mu^2 = 6.25$ or $\frac{25}{4}$ ③

using WAY 2:

...first option 1 :

$$\underline{I} = m(\underline{v} - \underline{u})$$

$$\begin{pmatrix}\lambda\\\mu\end{pmatrix} = 0.5\left(\begin{pmatrix}2a\\0\end{pmatrix} - \begin{pmatrix}4\\4\end{pmatrix}\right)$$

$$\Rightarrow \begin{pmatrix}\lambda\\\mu\end{pmatrix} = 0.5\begin{pmatrix}2a-4\\-4\end{pmatrix}$$

factor the 0.5 into bracket

$$\Rightarrow \begin{pmatrix}\lambda\\\mu\end{pmatrix} = \begin{pmatrix}a-2\\-2\end{pmatrix}$$

$\Rightarrow \lambda = a - 2$ -①

$\mu = -2$ -②

and using fact that the magnitude of $\underline{I}$ is 2.5 Ns (Pythagoras')

$$(\lambda)^2 + (\mu)^2 = (2.5)^2$$

$\Rightarrow \lambda^2 + \mu^2 = 6.25$ or $\frac{25}{4}$ ③

Question 3 continued

here, either subbing ② into ① for λ:

$$\frac{25}{4} = \lambda^2 + (-2)^2$$

$$\Rightarrow \frac{25}{4} = \lambda^2 + 4$$

$-4 \qquad -4$

$$\lambda^2 = \frac{9}{4}$$

square root

$$\Rightarrow \lambda = \pm 3/2$$

∴ IMPULSE for ~~option 1~~ is either $\boxed{\begin{pmatrix} 3/2 \\ 2 \end{pmatrix} \text{Ns or} \begin{pmatrix} -3/2 \\ 2 \end{pmatrix} \text{Ns}}$

OR subbing ① and ② into ③:

$$(0.5a - 2)^2 + (-2)^2 = 25/4$$

$-4 \qquad -4$

$$\Rightarrow (0.5a - 2)^2 = 9/4$$

square root OR expanding and solving the resulting quadratic:

…square root:

$$0.5a - 2 = \pm\sqrt{9/4} = \pm 3/2$$

⊕ $0.5a - 2 = 3/2$ ⊖ $0.5a - 2 = -3/2$

$\Rightarrow \frac{1}{2}a = 7/2 \qquad \Rightarrow 0.5a = 1/2$

$\times 2 \quad \times 2 \qquad \div 0.5 \quad \div 0.5$

$\Rightarrow a = 7 \qquad \Rightarrow a = 1$

OR

…expanding quadratic:

$$0.25a^2 - 2a + 4 = 2.25$$

$$0.25a^2 - 2a + 1.75 = 0$$

$\div 0.25 (\times 4) \qquad \div 0.25 (\times 4)$

$$a^2 - 8a + 7 = 0$$

factorise:

$$(a-7)(a-1) = 0$$

$$\Rightarrow a = 7 \text{ or } a = 1$$

Subbing the a's into ①

$$\Rightarrow \lambda = 0.5(7) - 2 \qquad \mu = -2$$

$$\Rightarrow \lambda = 1.5 \text{ or } 3/2$$

$$\therefore \boxed{I = \begin{pmatrix} 3/2 \\ -2 \end{pmatrix}}$$

here, either subbing ② into ① for λ:

$$\frac{25}{4} = \lambda^2 + (-2)^2$$

$$\frac{25}{4} = \lambda^2 + 4$$

$-4 \qquad -4$

$$\lambda^2 = 9/4$$

square root:

$$\Rightarrow \lambda = \pm 3/2$$

∴ IMPULSE for ~~option 1~~ is either $\begin{pmatrix} 3/2 \\ 2 \end{pmatrix}$ Ns or $\begin{pmatrix} -3/2 \\ 2 \end{pmatrix}$ Ns

OR subbing ① and ② into ③:

$$(a-2)^2 + (-2)^2 = 25/4$$

$$\Rightarrow (a-2)^2 = 9/4$$

square root OR expanding and solving the resulting quadratic

…square root:

$$a - 2 = \pm 3/2$$

⊕ $a - 2 = 3/2$ ⊖ $a - 2 = -3/2$

$+2 \quad +2$

$\Rightarrow a = 7/2 \qquad \Rightarrow a = 1/2$

OR

…expanding quadratic:

$$a^2 - 4a + 4 + 4 = 25/4$$

$$a^2 - 4a + 7/4 = 0$$

$\times 4 \qquad \times 4$

$$4a^2 - 16a + 7 = 0$$

calc equtn solver

$$\Rightarrow a = 7/2 \text{ or } 1/2$$

Subbing the a's into ①

$$\Rightarrow \lambda = (7/2) - 2$$

$$= 3/2 \qquad \boxed{\therefore I = \begin{pmatrix} 3/2 \\ -2 \end{pmatrix}}$$

Question 3 continued

...and into ②:

$\lambda = 0.5(1) - 2$; $\mu = -2$

$= -1.5$ or $-3/2$

$\therefore I = \begin{pmatrix} -3/2 \\ -2 \end{pmatrix}$ Ns

... and into ②:

$\lambda = (1/2) - 2$; $\mu = -2$

$= -3/2$

$\therefore I = \begin{pmatrix} -3/2 \\ -2 \end{pmatrix}$ Ns

...next option 2:

$I = m(v - u)$

$\begin{pmatrix} \lambda \\ \mu \end{pmatrix} = 0.5\left(\begin{pmatrix} 0 \\ b \end{pmatrix} - \begin{pmatrix} 4 \\ 4 \end{pmatrix}\right)$

$\Rightarrow \begin{pmatrix} \lambda \\ \mu \end{pmatrix} = 0.5\begin{pmatrix} -4 \\ b-4 \end{pmatrix}$

factor the 0.5 into bracket

$\begin{pmatrix} \lambda \\ \mu \end{pmatrix} = \begin{pmatrix} -2 \\ 0.5b-2 \end{pmatrix}$

$\Rightarrow \lambda = -2$ -①

$\mu = 0.5b - 2$ -②

...next, option 2 :

$I = m(v - u)$

$\begin{pmatrix} \lambda \\ \mu \end{pmatrix} = 0.5\left(\begin{pmatrix} 0 \\ 2b \end{pmatrix} - \begin{pmatrix} 4 \\ 4 \end{pmatrix}\right)$

$\Rightarrow \begin{pmatrix} \lambda \\ \mu \end{pmatrix} = 0.5\begin{pmatrix} -4 \\ 2b-4 \end{pmatrix}$

factor the 0.5 into the bracket

$\Rightarrow \begin{pmatrix} \lambda \\ \mu \end{pmatrix} = \begin{pmatrix} -2 \\ b-2 \end{pmatrix}$

$\Rightarrow \lambda = -2$ -①

$\mu = b - 2$ -②

and using fact that the magnitude of I is 2.5 Ns (Pythagoras')

$(\lambda)^2 + (\mu)^2 = (2.5)^2$

$\Rightarrow \lambda^2 + \mu^2 = 6.25 = \frac{25}{4}$ -③

here either sub ① into ③

$(-2)^2 + \mu^2 = 25/4$

-4 $\quad -4$

$\Rightarrow \mu^2 = 9/4$

square root:

$\mu = \pm 3/2$

$\therefore I = \begin{pmatrix} -2 \\ 3/2 \end{pmatrix}$ Ns or $\begin{pmatrix} -2 \\ -3/2 \end{pmatrix}$ Ns

and using fact that the magnitude of I is 2.5 Ns (Pythagoras')

$(\lambda)^2 + (\mu)^2 = (2.5)^2$

$\Rightarrow \lambda^2 + \mu^2 = 6.25 = 25/4$ -③

here either sub ① into ③:

$(-2)^2 + \mu^2 = 25/4$

$\Rightarrow \mu^2 = 9/4$

square root:

$\mu = \pm 3/2$

$\therefore I = \begin{pmatrix} -2 \\ 3/2 \end{pmatrix}$ Ns or $\begin{pmatrix} -2 \\ -3/2 \end{pmatrix}$ Ns

OR sub ① and ② into ③

$(-2)^2 + (0.5b - 2)^2 = 6.25$ or $25/4$

$(0.5b - 2)^2 = 9/4$

either square root or use quadratic formula

... square rooting:

$0.5b - 2 = \pm 3/2$

⊕ $0.5b - 2 = 3/2$

$\Rightarrow 0.5b = 7/2$

$\div 0.5$ $\quad \div 0.5$

$\Rightarrow b = 7$

⊖ $0.5b - 2 = -3/2$

$\Rightarrow 0.5b = 1/2$

$\div 0.5$ $\quad \div 0.5$

OR sub ① and ② into ③

$(-2)^2 + (b-2)^2 = 25/4$

$\Rightarrow (b-2)^2 = 9/4$

either square root or use quadratic formula

... square rooting:

$b - 2 = \pm 3/2$

⊕ $b - 2 = 3/2$

$\Rightarrow b = 7/2$

⊖ $b - 2 = -3/2$

$\Rightarrow b = 1/2$

$= b = 1$

OR

...quadratic:

$0.25b^2 - 2b + 4 - 9/4 = 0$

$\frac{1}{4}b^2 - 2b + \frac{7}{4} = 0$

$\times 4 \quad b^2 - 8b + 7 = 0 \quad \times 4$

factorise

$(b-7)(b-1) = 0$

$\Rightarrow b = 7$ or $b = 1$

Subbing into ②

$a = 0.5(1) - 2 = -3/2$

$\therefore I = \begin{pmatrix} -2 \\ -3/2 \end{pmatrix}$ Ns

or

$a = 0.5(7) - 2$
$= 3/2$

$I = \begin{pmatrix} -2 \\ -3/2 \end{pmatrix}$ Ns

OR quadratic:

$b^2 - 4b + 4 = 9/4$

$\Rightarrow b^2 - 4b + 7/4 = 0$

$\times 4 \quad 4b^2 - 16b + 7 = 0 \quad \times 4$

calc equtn solver:

$\Rightarrow b = 7/2$ or $1/2$

Sub each 'b' into ②

$a = (7/2) - 2$
$= 3/2$

or

$a = (\frac{1}{2}) - 2$
$= -3/2$

$\therefore I = \begin{pmatrix} -2 \\ 3/2 \end{pmatrix}$ Ns or $\begin{pmatrix} -2 \\ -3/2 \end{pmatrix}$ Ns

$\therefore$ the 4 options:

$\begin{pmatrix} 3/2 \\ -2 \end{pmatrix}$ Ns, $\begin{pmatrix} -3/2 \\ -2 \end{pmatrix}$ Ns, $\begin{pmatrix} -2 \\ 3/2 \end{pmatrix}$ Ns, $\begin{pmatrix} -2 \\ -3/2 \end{pmatrix}$ Ns

(Total for Question 3 is 9 marks)

4. A car of mass 600 kg pulls a trailer of mass 150 kg along a straight horizontal road. The trailer is connected to the car by a light inextensible towbar, which is parallel to the direction of motion of the car. The resistance to the motion of the trailer is modelled as a constant force of magnitude 200 N. At the instant when the speed of the car is $v\,\text{m s}^{-1}$, the resistance to the motion of the car is modelled as a force of magnitude $(200 + \lambda v)$ N, where λ is a constant.

When the engine of the car is working at a constant rate of 15 kW, the car is moving at a constant speed of $25\,\text{m s}^{-1}$

(a) Show that $\lambda = 8$

(4)

Later on, the car is pulling the trailer up a straight road inclined at an angle θ to the horizontal, where $\sin\theta = \frac{1}{15}$

The resistance to the motion of the trailer from non-gravitational forces is modelled as a constant force of magnitude 200 N at all times. At the instant when the speed of the car is $v\,\text{m s}^{-1}$, the resistance to the motion of the car from non-gravitational forces is modelled as a force of magnitude $(200 + 8v)$ N.

The engine of the car is again working at a constant rate of 15 kW.

When $v = 10$, the towbar breaks. The trailer comes to instantaneous rest after moving a distance d metres up the road from the point where the towbar broke.

(b) Find the acceleration of the car immediately after the towbar breaks.

(4)

(c) Use the work-energy principle to find the value of d.

(4)

(a) let's illustrate the above information on a DETAILED FORCE DIAGRAM:

...label the RESISTANCE, the TENSION and the POWER rearranged:

$P = Fv \rightarrow$ VELOCITY in M/S^{-1}

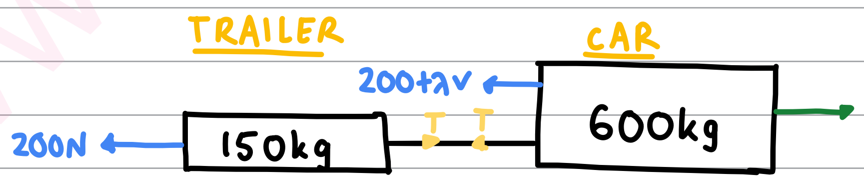
POWER in WATTS FORCE in NEWTONS

$\Rightarrow F = \frac{P}{V}$

convert into watts

$P = 15kW \xrightarrow{\times 1000} 15{,}000W$

and $V = 25$



$\underbrace{\frac{15{,}000W}{25ms^{-1}}}$ $= 600N$

NOTE: could've calculated this power as a separate line of working but much better in exams to straight away write in the FORCE from the formula rearranged (saves time)

Question 4 continued

the question is asking for the value of 'λ', hence we need to resolve to the right, bearing in mind from Yr 1 Mechanics Chp 8 that the fact that towbar is 'light' means that the tension is equal throughout ($\therefore$ can be ignored)

$$R(\rightarrow): \; 600-(200)-(200+\lambda v)=0$$

substituting v=25 as noticing we've got an expression for variable resistance

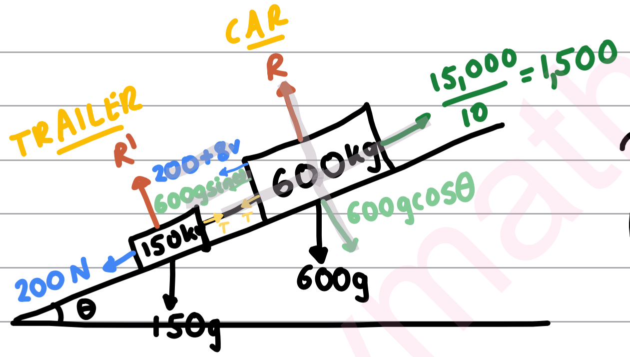
$$600-200-200-25\lambda=0$$

$$\Rightarrow 25\lambda = 200$$
$$\div 25 \qquad \div 25$$

$$\boxed{\lambda = 8}$$

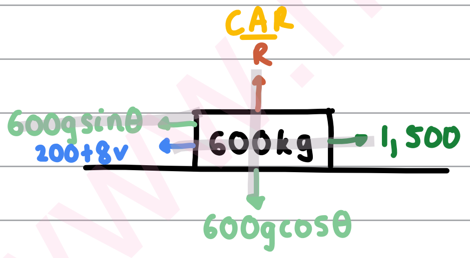
(b) let's look at the forces again - REDRAWING the part (a) diagram but on an inclined plane - label the RESISTANCE, the TENSION, the POWER rearranged

$$P=Fv$$
$$\Rightarrow F=\frac{P}{v}$$
$$15kW \xrightarrow{\times 1000} 15{,}000W$$



the question only focuses on finding the acceleration of the car - know from Chp 2 FM1 OR Chp 5 Mechanics Yr 2 - this would require us to resolve parallel to the plane

turn axis
(but only need CAR's acceleration so focus on this)



now as it's accelerating, use $\Sigma F_x = ma$

$$1{,}500-(600g\sin\theta)-(200+8v)=600(a)$$

subbing in v=10 to get 'a' after towbar breaks

$$1{,}500-600g\left(\tfrac{1}{15}\right)-200-8(10)=600a$$

$$\Rightarrow 1{,}220-40g=600a$$

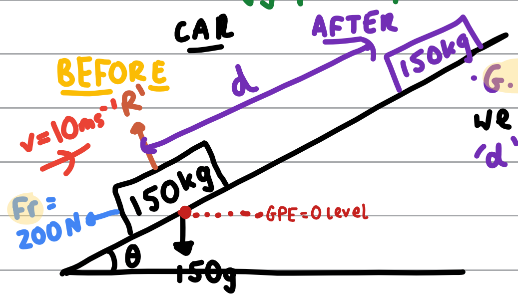
$$\Rightarrow 1{,}200-40(9.8)=600a$$

$$\Rightarrow 828=600a$$
$$\div 600 \qquad \div 600$$

$$\boxed{\Rightarrow a = \frac{828}{600} = 1.38\,ms^{-2}}$$

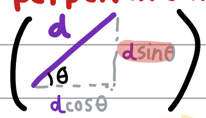
Question 4 continued

(c)... now focusing just on the TRAILER - re-drawing the diagram and adapting it to 'work-energy principle' - labelling the energies in:



· only E_k

· G.P.E (travelled 'd' m UP THE PLANE) - but remember we need to use the perpendicular component of 'd' so resolving 'd'



· work done against FRICTION (i.e the resistive force)

subbing into 'work energy' principle (Yr 1)

$$w.d\ in + K.E_i + G.P.E_i = K.E_f + G.P.E_f + w.d \text{ against friction}$$

initial kinetic; initial grav. potential; final kinetic; final grav. potential

$$\underbrace{\left(F = \frac{P}{v} \times d\right)} + \frac{1}{2}mu^2 + mgh_1 = \frac{1}{2}mv^2 + mgh_2 + f_r \times d$$

NON APPLICABLE as towbar broke ∴ no engine force from car

$$\frac{1}{2}(150)(10)^2 + 0 = 0 + 150g(d\sin\theta) + 200(d)$$

$$\Rightarrow 7{,}500 = 150(g)(d/15) + 200d$$

$$\Rightarrow 7{,}500 = 10(9.8)d + 200d$$

$$\Rightarrow 298d = 7{,}500$$

$$\div 298 \qquad \div 298$$

$$\boxed{\Rightarrow d = 25.16778\ldots = 25.2\ (3\text{ s.f})}$$

Question 4 continued

(Total for Question 4 is 12 marks)

5. A particle P of mass $3m$ and a particle Q of mass $2m$ are moving along the same straight line on a smooth horizontal plane. The particles are moving in opposite directions towards each other and collide directly.

Immediately before the collision the speed of P is u and the speed of Q is $2u$.

Immediately after the collision P and Q are moving in opposite directions.

The coefficient of restitution between P and Q is e.

(a) Find the range of possible values of e, justifying your answer.

(8)

Given that Q loses 75% of its kinetic energy as a result of the collision,

(b) find the value of e.

(3)

(a) illustrating this linear collision diagrammatically - illustrating respective speeds, direction of motion etc.

NOTE: for the rest of the question we're going to be using these directions ∴ straightaway $x, y > 0$ (will use later!)

and following the usual procedure for elastic collisions in 1D notice how both speeds after are unknown ∴ can't stop at just using PCLM - need to do NEL (Impact law) as well:

...first PCLM - means the total momentum before the collision equals the total momentum after:

formula: $m_P u_P + m_Q u_Q = m_P v_P + m_Q v_Q$

$3m(u) + 2m(-2u) = 3m(-x) + 2m(y)$

expand brackets

$3\cancel{m}u - 4\cancel{m}u = -3\cancel{m}x + 2\cancel{m}y$

$\Rightarrow -3x + 2y = -u \quad \times -1$

$\Rightarrow 3x - 2y = u \quad -①$

...now NEL - i.e formula to find coefficient of restitution:

formula: $e = \dfrac{\text{speed of separation}}{\text{speed of approach}} = \dfrac{v_Q - v_P}{u_P - u_Q}$

subbing into above

$e = \dfrac{y - (-x)}{u - (-2u)} = \dfrac{y + x}{3u}$

$\times 3u$ $\times 3u$

Question 5 continued

$3eu = y + x$
$\Rightarrow y + x = 3eu$ — ②

solve ① and ② simultaneously – elim. 'y':

① − 2×②

$3x - 2y = u$
$+\ 2x + 2y = 6eu$
$5x = u + 6eu$

factorise 'u' on RHS

$5x = u(1 + 6e)$
$\div 5$ $\div 5$
$\boxed{x = \frac{u}{5}(1 + 6e)}$

next elim. 'x':

3×② − ①

$3y + 3x = 9eu$
$+\ 2y - 3x = -u$
$5y = 9eu - u$

factorise 'u' on RHS

$5y = u(9e - 1)$
$\div 5$ $\div 5$
$\Rightarrow \boxed{y = \frac{u}{5}(9e - 1)}$

now for RANGES for 'e' – need to exploit those facts about x and y that we spoke about earlier – that $x, y > 0$

...for 'x':

$x = \frac{u}{5}(1 + 6e) > 0$
$\div u/5$ $\div u/5$
$1 + 6e > 0$
$6e > -1$
$\div 6$ $\div 6$
$e > -1/6$
but $0 \leq e \leq 1$ so reject

...for 'y':

$y = \frac{u}{5}(9e - 1) > 0$
$\div u/5$ $\div u/5$
$9e - 1 > 0$
$9e > 1$
$\div 9$ $\div 9$
$e > 1/9$ which fits $0 \leq e \leq 1$ ∴ include in interval

∴ combining above facts about 'e'

$\Rightarrow 1/9 < e \leq 1$

(b) know that this is the 'kinetic energy' part of Chp 4
↳ if Q loses 75% of its $K.E_i$, this suggests that $K.E_{final} = 25\%(1/4)$ of $(\times)$ its $K.E_{init.}$
remembering the formula for: $K.E\ initial = \frac{1}{2}m(u_Q)^2$

Question 5 continued

K.E final $= \frac{1}{2}m(v_Q)^2$

subbing into formulae and the equation :

$$\frac{1}{2}(2m)\left(\frac{u}{5}(9e-1)\right)^2 = \frac{1}{4} \times \frac{1}{2}(2m)(2u)^2$$

expand brackets

$$mu^2\left(\frac{1}{25}(9e-1)^2\right) = \frac{1}{4}(4mu^2)$$

$$\Rightarrow \frac{\cancel{mu^2}}{25}(9e-1)^2 = \cancel{mu}^2$$

$\times 25$ $\times 25$

$$(9e-1)^2 = 25$$

and solve above for the value of 'e'

WAY 1: square root	WAY 2: expand and solve quadratic
$9e-1 = \pm 5$ $\oplus$ $9e-1=5$ ⇒ $9e=6$, $\div 9$ $\div 9$, $\boxed{e = 6/9 = 2/3}$ or $\ominus$ $9e-1=-5$ ⇒ $9e=-4$, $\div 9$ $\div 9$, $\boxed{e=-4/9}$	$81e^2 - 18e + 1 = 25$ $\Rightarrow 81e^2 - 18e - 24 = 0$ calc equtn solver $\boxed{\Rightarrow e = 2/3 \text{ or } -4/9}$

but bearing in mind that $\frac{1}{9} < e \leq 1$ from part (a)

$$\boxed{\therefore e = 2/3}$$

Question 5 continued

(Total for Question 5 is 11 marks)

6. [*In this question* **i** *and* **j** *are perpendicular unit vectors in a horizontal plane.*]

A smooth uniform sphere A has mass 0.2 kg and another smooth uniform sphere B, with the same radius as A, has mass 0.4 kg.

The spheres are moving on a smooth horizontal surface when they collide obliquely. Immediately before the collision, the velocity of A is $(3\mathbf{i} + 2\mathbf{j})\,\text{m s}^{-1}$ and the velocity of B is $(-4\mathbf{i} - \mathbf{j})\,\text{m s}^{-1}$

At the instant of collision, the line joining the centres of the spheres is parallel to **i**

The coefficient of restitution between the spheres is $\frac{3}{7}$

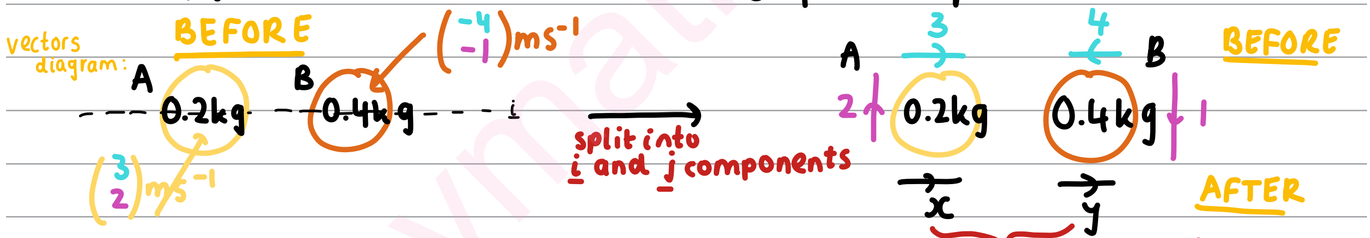
(a) Find the velocity of A immediately after the collision. **(7)**

(b) Find the magnitude of the impulse received by A in the collision. **(2)**

(c) Find, to the nearest degree, the size of the angle through which the direction of motion of A is deflected as a result of the collision. **(3)**

(a) notice now we have an 'oblique collisions between two spheres' question - first illustrating the collision with a diagram:

... label: · velocities direction · line of centres parallel to i



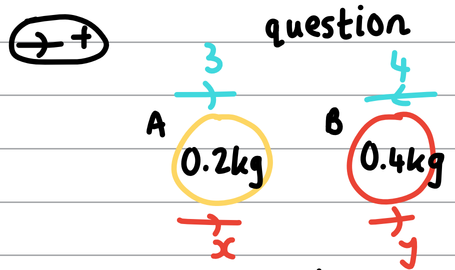
NOTE: initially easier to aim 'x' and 'y' RIGHTWARDS to avoid -ves

now remembering how as two spheres collide obliquely, the IMPACT (impulse) acts ALONG THEIR LINE OF CENTRES, which implies that:

... parallel components:

final velocities change - becomes a standard 'collisions in 1D' question

(→ +)



notice how because both final velocities are unknown, have to use PCLM AND NEL (Impact law)

... first PCLM: states that momentum BEFORE collision equals momentum AFTER

Question 6 continued

formula: $m_A u_A + m_B u_B = m_A v_A + m_B v_B$

substituting into formula

$0.2(3) + 0.4(-4) = 0.2(x) + 0.4(y)$

expand brackets

$0.2x + 0.4y = -1$ – ①

... next NEL – i.e formula for coefficient of restitution:

$$e = \frac{\text{speed of separation}}{\text{speed of approach}} = \frac{v_B - v_A}{u_A - u_B}$$

$$\Rightarrow \frac{3}{7} = \frac{y - x}{3 - (-4)}$$

$$\Rightarrow \frac{3}{7} = \frac{y - x}{7}$$

equating numerators

$\Rightarrow y - x = 3$ – ②

solving ① and ② simultaneously – calc equtn solver or elim. y (question asks for 'x')

① – 2×②

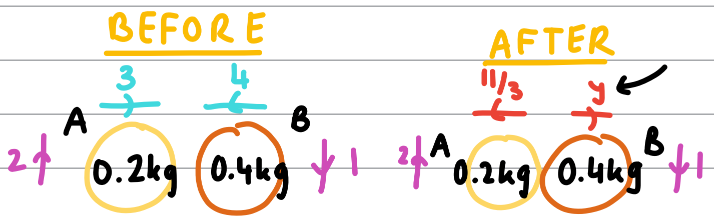
$$\begin{aligned} & x + 2y = -5 \\ - \; & -2x + 2y = 6 \\ \hline & 3x = -11 \\ & \div 3 \quad \div 3 \end{aligned}$$

$\Rightarrow x = -11/3$

here the negative suggests that particle A did in fact go LEFTWARDS

... perpendicular components: REMAIN THE SAME

hence populating initial diagram:



NOTE: no need in exam to try work out 'y' when not asked for v_B, just v_A

$$\boxed{\Rightarrow v_B = \begin{pmatrix} -11/3 \\ 2 \end{pmatrix} \text{ms}^{-1} \text{ or in } \underline{i}\text{-}\underline{j} \text{ notation: } \left(-\tfrac{11}{3}\underline{i} + 2\underline{j}\right)\text{ms}^{-1}}$$

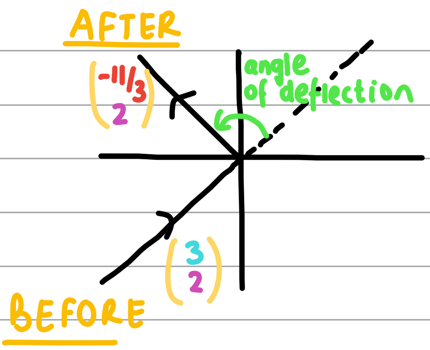
Question 6 continued

(b) now that we have both the initial and final parallel components of A's velocity - can work out the IMPULSE we mentioned in part (a) through subbing into Impulse-momentum principle: $I = m(v-u)$ (after all, impulse only acts parallel to the line of centres)

$$I = 0.2\left(-\tfrac{11}{3} - 3\right)$$

$$\boxed{\Rightarrow I = \tfrac{4}{3}\ \text{Ns}}$$

(c) representing A's path diagrammatically:



... need to find this angle of deflection:

METHOD 1: we will use the formula for angle between 2 vectors:

$$\cos\theta = \frac{a \cdot b}{|a||b|}$$

scalar product

magnitude

sub into above

$$\cos\theta = \frac{u \cdot v}{|u||v|}$$

$$\Rightarrow \cos\theta = \frac{\begin{pmatrix} 3 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} -11/3 \\ 2 \end{pmatrix}}{\sqrt{(3)^2+(2)^2}\sqrt{(-11/3)^2+(2)^2}} = \frac{3(-11/3)+2(2)}{\sqrt{13}\sqrt{157/9}}$$

$$= \frac{-11+4}{\sqrt{13}\sqrt{157/9}} = \frac{-7}{\sqrt{13}\sqrt{157/9}}$$

but need the angle:

$$\theta = \cos^{-1}\left(\frac{-7}{\sqrt{13}\sqrt{157/9}}\right)$$

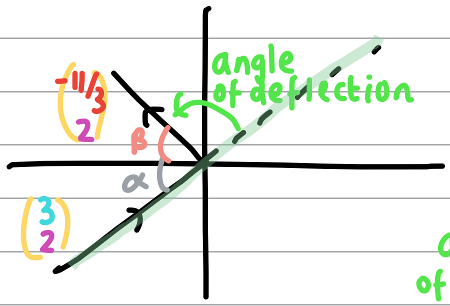
evaluate on CALC (in degrees mode)

$$\Rightarrow \theta = 117.69947\ldots$$

$$\boxed{= 118^\circ\ (3\text{s.f})}$$

METHOD 2: considering the two vectors SEPARATELY - making 'α' and 'β' to the x-axis

Question 6 continued



see from this diagram that can exploit 'angles on a straight line' properties

here the direction doesn't matter

$$\text{angle of deflection} = 180° - \tan^{-1}\left(\frac{2}{3}\right) - \tan^{-1}\left(\frac{2}{11/3}\right)$$

$$= 180° - \tan^{-1}(2/3) - \tan^{-1}(6/11)$$

$$= 117.699.. \boxed{= 118° \text{ (3 s.f)}}$$

(Total for Question 6 is 12 marks)

7. A particle P, of mass m, is attached to one end of a light elastic spring of natural length a and modulus of elasticity kmg.

The other end of the spring is attached to a fixed point O on a ceiling.

The point A is vertically below O such that $OA = 3a$

The point B is vertically below O such that $OB = \frac{1}{2}a$

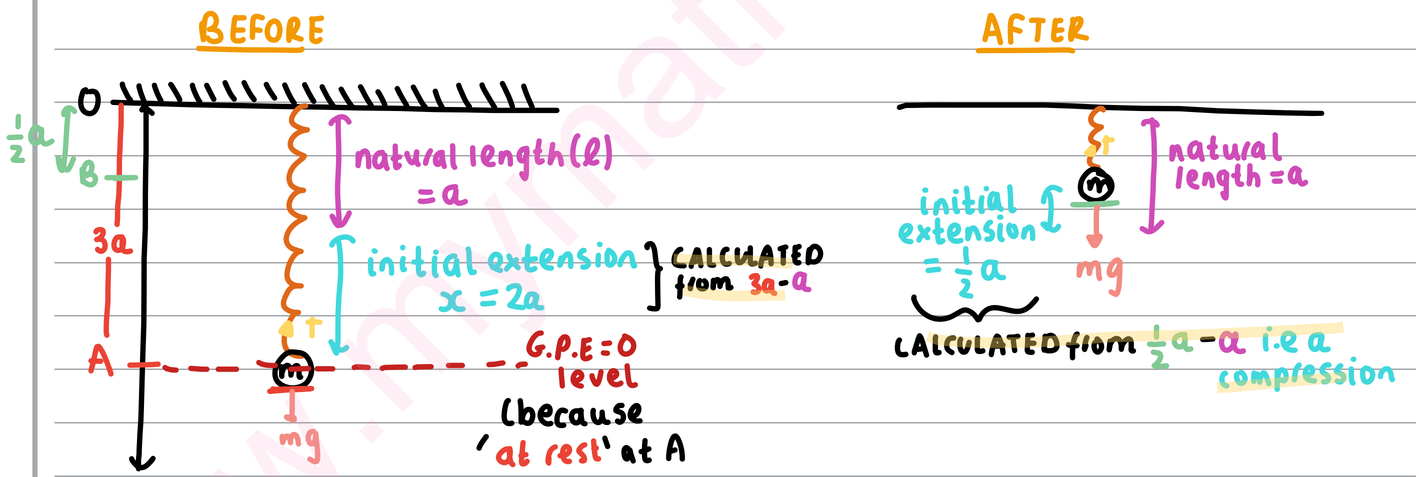
The particle is held at rest at A, then released and first comes to instantaneous rest at the point B.

(a) Show that $k = \frac{4}{3}$
(3)

(b) Find, in terms of g, the acceleration of P immediately after it is released from rest at A.
(3)

(c) Find, in terms of g and a, the maximum speed attained by P as it moves from A to B.
(6)

(a) always with 'elastic strings and springs' questions, have to focus on drawing the right diagram –...label:
- spring (not string) – $\lambda = kmg$
- $l = a$



· E.P.E (stretched)

· E.P.E (compressed AND a spring - NOT a string that can't be compressed)
· G.P.E (travelled through a distance: A − B i.e $3a - \frac{1}{2}a = \frac{5}{2}a$)

using the conservation of mechanical energy principle - the total amount of MECHANICAL ENERGY (K.E, G.P.E, E.P.E) in a closed system, in the absence of dissipative forces (friction, air resistance) remains constant

formula : $K.E_i + G.P.E_i + E.P.E_i = K.E_f + G.P.E_f + E.P.E_f$

initial kinetic, initial grav. potential, initial elastic potential, final kinetic, final grav. potential, final elastic potential

Question 7 continued

$$\frac{1}{2}mu^2 + mgh_1 + \frac{\lambda x^2}{2l} = \frac{1}{2}mv^2 + mgh_2 + \frac{\lambda x^2}{2l}$$

substitute in

$$0 + 0 + \frac{kmg(2a)^2}{2(a)} = 0 + mg(5/2\,a) + \frac{kmg(\frac{1}{2}a)^2}{2(a)}$$

released 'from rest' → 0; because of GPE = 0 level → 0; comes to 'instantaneous rest' → 0

expand + cancel 'a's and 'mg's

$$\frac{\cancel{mg}k(4a^2)}{2\cancel{a}} = \frac{\frac{1}{4}k\cancel{mg}a^2}{2\cancel{a}} + \frac{5}{2}\cancel{mg}(a)$$

$$\Rightarrow 2k\cancel{a} = \frac{1}{8}k\cancel{a} + \frac{5}{2}\cancel{a}$$

collect like 'k' terms

$$\frac{15}{8}k = 5/2$$

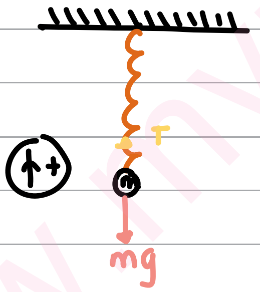
$\div 15/8$ $\quad$ $\div 15/8$

$$\Rightarrow k = \frac{\cancel{5}^{1}}{2} \times \frac{\cancel{8}^{4}}{\cancel{15}_{3}} = \frac{4}{3}$$

$$\boxed{\therefore k = 4/3}$$

(b) now asked to evaluate a DYNAMICS springs question:

HINTS at 2nd Newton's law – $\Sigma F = ma$ – drawing a FORCE DIAGRAM for the spring:



determining upwards as +ve (because spring going UPWARDS to stop at B)

R(↑)

$$T - mg = ma$$

where $T = \frac{\lambda x}{l}$ for STRINGS/SPRINGS

subbing in and cancelling

$$\frac{\text{"}4/3\text{"}\,mg(2\cancel{a})}{\cancel{a}} - mg = ma$$

$$\frac{8}{3}\cancel{m}g - \cancel{m}g = \cancel{m}a$$

collect like terms

$$\Rightarrow \frac{5}{3}mg = a$$

$$\boxed{\therefore a = 5/3\,mg\ (ms^{-2})}$$

(c) recall from Yr 1 Mechanics that the max. speed for any object in motion

Question 7 continued

occurs when $a=0$ ∴ in FMI this would mean when the spring reaches equilibrium (call this Q)

↳ this occurs when $T=mg$ (from FORCE DIAGRAM in (b))

subbing tension formula in:

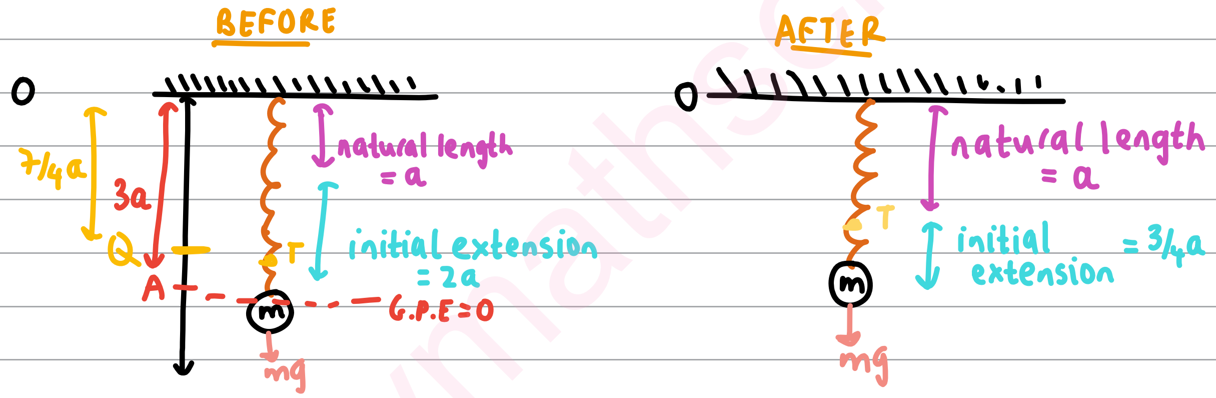
$$\frac{\lambda x}{l} = mg$$

$$\frac{\frac{4}{3}\cancel{mg}(x)}{a} = \cancel{mg} \quad \times a \quad \times a$$

$$\Rightarrow \frac{4}{3}x = a \quad \div 4/3 \quad \div 4/3$$

$$\Rightarrow x = 3/4 a \quad (\therefore OQ = l + x = a + 3/4 a = 7/4 a$$

so now we know the extension at the equilibrium position, we can use the work-energy principle from A to Q to find this max. speed



· E.P.E (stretched)

· E.P.E (compressed)

· G.P.E (travelled through a distance - OA − OE = $3a - 7/4 a = 5/4 a$)

subbing this into conservation of mech. energy formula: (can get max speed from the 'v' in $\frac{1}{2}mv^2$)

$$\frac{1}{2}mu^2 + mgh_1 + \frac{\lambda x^2}{2l} = \frac{1}{2}mv^2 + mgh_2 + \frac{\lambda x^2}{2l}$$

$$0 + 0 + \frac{\frac{4}{3}mg(2a)^2}{2(a)} = \frac{1}{2}mv^2 + mg(5/4 a) + \frac{\frac{4}{3}mg\left(\frac{3a}{4}\right)^2}{2(a)}$$

...simplifies to:

$$\frac{8}{3}ag = \frac{1}{2}v^2 + \frac{5}{4}ag + \frac{3}{8}ag$$

(Total for Question 7 is 12 marks)

TOTAL FOR PAPER IS 75 MARKS

collect like 'ag' terms:

$$\frac{1}{2}v^2 = \frac{25}{24}ag$$

$$\Rightarrow v^2 = \frac{25}{12}ag$$

square root:

$$\boxed{v = \sqrt{\frac{25}{12}ag} \text{ or } \frac{5}{2}\sqrt{ga/3}}$$